4. Hausübung, **Statistische Physik**

abzugeben am Donnerstag, 10.11.2011

Aufgabe H7 *Gibbs' paradox* (5 Punkte)

Recall that the partition function for one free particle of mass M in a box of volume N is

$$Z_1 = n_Q(\tau)V$$

where

$$n_Q(\tau) = \left(\frac{M\tau}{2\pi\hbar^2}\right)^{\frac{3}{2}} .$$

a. Consider N independent such particles, and assume that they are *distinguishable*. Show that the entropy is

$$\sigma(N, V) = N \ln(n_Q(\tau)V) + \frac{3}{2}N.$$

b. Consider two identical boxes of volume V at temperature τ, each containing a gas of N distinguishable particles, and compute the total entropy for this system. Show that the difference between the total entropy for the two separate boxes and the total entropy for a single box of volume $2V$ containing $2N$ particles is

$$\Delta\sigma = -2N \ln 2.$$

This means that the entropy can be increased or decreased by a macroscopic amount just by inserting a barrier in the center of the large box or removing it, which contradicts the principles of thermodynamics.

c. Do the same calculation when the particles are indistinguishable and show that, for $N \gg 1$,

$$\Delta\sigma \simeq 0.$$

(more problems on the next page..)

Aufgabe H8 *Gas in a potential* (4 Punkte)

Consider a classical particle with phase space coordinates (x, p) and Hamiltonian

$$H(x,p) = t(p) + v(x)$$

where t and v are real functions. Also we assume that the position x is restricted to a volume V. For the purpose of computing entropies or partition functions, we will count one state per phase space volume h. Let Z_1 be the canonical partition function of that system at temperature τ in the case where $v(x) = 0$.

a. Allowing for more than one particle, and assuming the particles do not interact and are *indistinguishable*, show that the grand-canonical partition function at temperature τ and with fugacity $\lambda = e^{\mu/\tau}$ is

$$\mathcal{Z} = e^{\frac{\lambda Z_1}{V} \int e^{-v(x)/\tau} dx}.$$

b. Show that the average number of particles is then

$$\langle N \rangle = \frac{Z_1}{V} \int e^{(\mu - v(x))/\tau} dx.$$

We will use this in the next problem.

Aufgabe H9 *Centrifuge* (3 Punkte)

Consider a circular cylinder of radius R and length L, rotating about its symmetry axis with angular velocity ω and containing an ideal gas with particles of mass M. We assume that the system is in thermal equilibrium at temperature τ, that the gas is at rest in a reference frame rotating with the cylinder, and that the particles' velocities are small enough that we can ignore all the Coriolis forces.

If $n(0)$ is the expected particle density on the axis of rotation, show that the expected particle density at a distance r is

$$n(r) = n(0)\, e^{\frac{1}{2}M\omega^2 r^2/\tau}.$$

Hint: Consider a small volume of gas a distance r from the axis. Observe that it is at thermal and diffusive equilibrium with the rest of the gas and subject to a centrifugal potential $v(r)$. Then use the result of problem H8, recalling that the partition function for one particle of mass M in a box of volume V and at temperature τ has the form

$$Z_1 = \left(\frac{\tau}{\alpha}\right)^{\frac{3}{2}} V,$$

for some fixed energy $\alpha > 0$.